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Rezumatul tezei de doctorat-engleză

Stabilization of the Navier-Stokes equations

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Preface

The mathematical model which describes the evolution of a fluid is one of the most studied by the researchers because of its major importance. The governing equations are the well-known Navier-Stokes equations.

In the first chapter, we study the evolution of a fluid in a semi-infinte rectangel. Main results obtained concerning the stabilization of the Poiseuille parabolic profile were obtained by Krstic and his cowrokers in [3, 85, 54, 85, 27, 77, 64]. We obtain a stabilizing feedback controller which acts only on the normal component of the velocity field, on the upper wall. The stability is guaranteed without any a priori condition on the viscosity coeffiecient. The controller is easily manageable from computational point of view since the operators in its form satisfy Riccati algebraic equations associated to parabolic equations on (0,1) with the same structure for diferent values of the Fourier modes.

The second chapter propose an nonlinear internal control for the Navier-Stokes equations with slip-boundary conditions. Moreover, we show that this controller steers the initial data into the space considting of stable modes, in finite time.

Finally, the last chaper study the problem in finding a feedback controller which stabilizes, not only one steady-state solution, but a finite-set of steady-state solutions for the Navier-Stokes equations, by using the allready known results obtained in [**?**]

The paper ends with an Apendix which contains a short introduction in the theoretical results used during the presentation.

Chapter 1

Stabilization of Navier-Stokes equations in a channel

In this chapter, we study the mathematical model which describes a Newtonian incompressible fluid evolving in a semi-infinte 2-D channel

$$(x, y) \in (-\infty, +\infty) \times (0, 1),$$

and 3-D channel

$$(x, y, z) \in (-\infty, +\infty) \times (0, 1) \times (-\infty, +\infty),$$

respectively. We shall consider both cases, when there is no action of magnetic type and the case when an external constant magnetic field acts transversal on the channel. The equations which govern the dynamics are the Navier-Stokes equations, and the MHD equations, a combination between the Navier-Stokes equations and Maxwell equations.

The both experimental and numerical analysis show that, for high values of Reynolds number (that is, for low values of the viscosity coefficient) the fluid may develop chaotic and turbulent movements. One of the principal mathematical tools used in order to atenuate or even eliminate the turbulence is the stabilization of the Navier-Stokes equations. In this chapter, we shall design a stabilizing feedback controller, which acts on the upper wall, on the normal component of the velocity field or on the tangential one. The stability is achived without any a priori assumptions on the Reynolds number. We shall use the method of Fourier decomposition of the linearized system and reduction of the pressure, developed by Barbu in [14], to obtain an inifinite parabolic system. The stabilization is achived at each level, by using the decomposition method of the system in the stable and unstable part, developed by Barbu and Triggiani in [10]. The feedback form of the stabilizing controller is obtained via minimization of a quadratic functional cost associated to a linear parabolic problem.

1.1 Normal feedback stabilization of periodic flows in a two-dimensional channel

This section entirely contains the original results obtained by the author in [66].

1.1.1 Presenting the problem

The 2-D Navier-Stokes equations in a periodic channel are given by

$$\begin{cases} u_t - \nu\Delta u + uu_x + vu_y = p_x, \\ v_t - \nu\Delta v + uv_x + vv_y = p_y, \\ u_x + v_y = 0, \\ \forall t \geq 0,\ x \in \mathbb{R},\ y \in (0,1), \\ u(t,x+2\pi,y) = u(t,x,y),\ v(t,x+2\pi,y) = v(t,x,y),\ p(t,x+2\pi,y) = p(t,x,y), \\ \forall t \geq 0,\ \forall x \in \mathbb{R},\ \forall y \in (0,1), \\ u(t,x,0) = u(t,x,1) = 0,\ v(t,x,0) = 0,\ v(t,x,1) = \Psi(t,x),\ \forall t \geq 0,\ \forall x \in \mathbb{R}, \end{cases} \tag{1.1.1}$$

and initial data

$$u(0,x,y) = u_o(x,y),\ v(0,x,y) = v_o(x,y),\ \forall x \in \mathbb{R},\ \forall y \in (0,1).$$

Here $u = u(t,x,y)$ and $v = v(t,x,y)$ are the tangential component, normal component respectively, of the fluid; $p = p(t,x,y)$ is the pressure and ν is the viscosity coefficient.

The equilibrium solution which will be stabilized is the parabolic Poiseuille profile, given by

$$U^e(y) = C(y^2 - y),\ V^e \equiv 0,$$

where $C = -\frac{a}{2\nu}$, $a \in \mathbb{R}_+$.

The linearization of (1.1.1) around the steady-state is given by

$$\begin{cases} u_t - \nu\Delta u + u_x U^e + vU^e_y = p_x, \\ v_t - \nu\Delta v + v_x U^e = p_y, \\ u_x + v_y = 0, \\ u(t,x,0) = u(t,x,1) = 0,\ v(t,x,0) = 0,\ v(t,x,1) = \Psi(t,x), \\ u(t,x+2\pi,y) = u(t,x,y),\ v(t,x+2\pi,y) = v(t,x,y), \\ p(t,x+2\pi,y) = p(t,x,y),\ \forall t \geq 0,\ x \in \mathbb{R},\ y \in (0,1), \end{cases} \tag{1.1.2}$$

and initial data

$$u(0,x,y) = u^0(x,y) := u_o(x,y) - U^e(y),\ v(0,x,y) = v^0(x,y) := v_o(x,y),$$
$$\forall x \in \mathbb{R},\ y \in (0,1).$$

In what follows we shall design a normal finite-dimensional feedback controller Ψ which stabilizes the linearized equation (1.1.2)

1.1.2 Preliminaries

The Fourier functional setting, in which we shall work, is given by the space $L^2_{2\pi}(Q), Q = (0,2\pi)\times(0,1)$, containig all the functions $u\in L^2_{loc}(\mathbb{R}\times(0,1))$, which are $2\pi-$periodic with respect to x. These functions are caracterized by their Fourier series

$$u(x,y)=\sum_{k\in\mathbb{Z}}u_k(y)e^{\mathrm{i}kx},\ u_k=\overline{u_{-k}},\forall k\in\mathbb{Z},$$

such that

$$\sum_{k\in\mathbb{Z}}\int_0^1|u_k(y)|^2dy<\infty.$$

The norm in $L^2_{2\pi}(Q)$ is

$$\|u\|_{L^2_{2\pi}(Q)}:=\left(\sum_{k\in\mathbb{Z}}2\pi\|u_k\|^2_{L^2(0,1)}\right)^{\frac{1}{2}}.$$

WE shall denote by $\|\cdot\|$ in both spaces $L^2_{2\pi}(Q)$ and $L^2(0,1)$.

We consider H to be the complexified space of $L^2(0,1)$. We denote by $\|\cdot\|$ the norm in H, and by $<\cdot,\cdot>$ the scalar product.

Returning to system (1.1.2) we rewrite it in terms of Fourier modes of the velocity field, the pressure and the control, that is

$$u=\sum_{k\in\mathbb{Z}}u_k(t,y)e^{\mathrm{i}kx},\ v=\sum_{k\in\mathbb{Z}}v_k(t,y)e^{\mathrm{i}kx}$$

and

$$p=\sum_{k\in\mathbb{Z}}p_k(t,y)e^{\mathrm{i}kx},\ \Psi=\sum_{k\in\mathbb{Z}}\psi_k(t)e^{\mathrm{i}kx}.$$

We obtain

$$\begin{cases}(u_k)_t-\nu u_k''+(\nu k^2+\mathrm{i}kU^e)u_k+(U^e)'v_k=\mathrm{i}kp_k \text{ a.p.t. în } (0,1),\\ (v_k)_t-\nu v_k''+(\nu k^2+\mathrm{i}kU^e)v_k=p_k' \text{ a.p.t. în } (0,1),\\ \mathrm{i}ku_k+v_k'=0 \text{ a.p.t. în } (0,1),\\ u_k(0)=u_k(1)=0, v_k(0)=0, v_k(1)=\psi_k,\end{cases}\tag{1.1.3}$$

and initial data u_k^0, v_k^0, for all $k\in\mathbb{Z}$. (We denote by $'$ the partial derivative with respect to y, i.e., $\frac{\partial}{\partial y}$.)

Next, the idea is to elimante the pressure from the equations in the next manner: we derive $(1.1.3)_1$ with respect to y and add the result to $(1.1.3)_2$ multiplied by ik. Using also the divergence free condition, we arrive at the next system, verified by v_k

$$\begin{cases}(-v_k''+k^2v_k)_t+\nu v_k''''-(2\nu k^2+\mathrm{i}kU^e)v_k''\\ \qquad\qquad +k(\nu k^3+\mathrm{i}k^2U^e+\mathrm{i}(U^e)'')v_k=0,\ \forall t\geq 0,\ \forall y\in(0,1),\\ \\ v_k'(t,0)=v_k'(t,1)=0, v_k(t,0)=0, v_k(t,1)=\psi_k(t),\ \forall t\geq 0,\\ \\ v_k(0,y)=v_k^0(y),\ y\in(0,1).\end{cases}\tag{1.1.4}$$

We introduce the operators

$$L_k : \mathcal{D}(L_k) \subset H \to H \text{ and } F_k : \mathcal{D}(F_k) \subset H \to H,$$

defined as

$$L_k v := -v'' + k^2 v,\ \mathcal{D}(L_k) = H^2(0,1) \cap H_0^1(0,1), \tag{1.1.5}$$

$$F_k v := \nu v'''' - (2\nu k^2 + \mathrm{i}kU^e)v'' + k(\nu k^3 + ik^2 U^e + \mathrm{i}(U^e)'')v, \tag{1.1.6}$$

$$\mathcal{D}(F_k) = H^4(0,1) \cap H_0^2(0,1).$$

AS well, we introduce the next differential forms

$$\mathcal{L}_k v := -v'' + k^2 v,$$

şi

$$\mathcal{F}_k v := \nu v'''' - (2\nu k^2 + \mathrm{i}kU^e)v'' + k(\nu k^3 + \mathrm{i}k^2 U^e + \mathrm{i}(U^e)'')v.$$

Then, system (1.1.4) can be written as

$$(L_k(v_k - w_k))_t + (F_k L_k^{-1})L_k(v_k - w_k) = \theta_k w_k - (\mathcal{L}_k(w_k))_t, \tag{1.1.7}$$

where $w_k = w_k(t,y)$ satisfies

$$\begin{cases} \theta_k w_k + \mathcal{F}_k w_k = 0,\ t \geq 0,\ y \in (0,1), \\ w_k'(0) = w_k'(1) = 0, w_k = 0, w_k(1) = \psi_k, \end{cases} \tag{1.1.8}$$

for some $\theta_k > 0$, sufficiently large. This suggests to introducce the next operators

$$\mathbf{A}_k : \mathcal{D}(\mathbf{A}_k) \subset H \to H, \text{ for all } k \in \mathbb{Z}^*,$$

defined as

$$\mathbf{A}_k := F_k L_k^{-1},\ \mathcal{D}(\mathbf{A}_k) = \left\{ v \in H : L_k^{-1} v \in \mathcal{D}(F_k) \right\}, \tag{1.1.9}$$

for which we have the next result, due to de Barbu [17].

Lema 1.1.1 *For all $k \in \mathbb{Z}^*$, operator $-\boldsymbol{A}_k$ generates a C_0 analytic semigroup in H, and for all $\lambda \in \rho(-\boldsymbol{A}_k)$, $(\lambda I + \boldsymbol{A}_k)^{-1}$ is compact. Besides, we have*

$$\sigma(-\boldsymbol{A}_k) \subset \{\lambda \in \mathbb{C} : \Re\lambda \leq 0\}, \forall |k| > S,$$

where

$$S = \frac{1}{\sqrt{\nu}} \left(1 + \frac{a}{\sqrt{2}\nu} \right)^{\frac{1}{2}}. \tag{1.1.10}$$

A first consequence of this lemma is: for all $|k| > S$, the solution to the system (1.1.4), with null control on the boundary, satisfies the next exponential decay

$$\|u_k(t)\|^2 + \|v_k(t)\|^2 \leq e^{-\nu S^2 t} \left(\|u_k^0\|^2 + \|v_k^0\|^2 \right),\ \forall t \geq 0,\ \forall |k| > S. \tag{1.1.11}$$

Therefore, it remains to control system (1.1.4) for $0 < |k| \leq S$ only.

After some computations it tourns out that system (1.1.7) can be written equivalently in the next form

$$\frac{d}{dt}(\tilde{L}_k v_k(t)) + \tilde{\mathbf{A}}_k(\tilde{L}_k(v_k)(t)) = (\theta_k + \tilde{F}_k)(D_k\psi_k(t)), t > 0. \tag{1.1.12}$$

Here $\tilde{L}_k$, $\tilde{F}_k$ and $\tilde{\mathbf{A}}_k$ are the extensions to H of the operators L_k, F_k, $\mathbf{A}_k$, respectively; and D_k is the Dirichlet operator associated to $\theta_k + \tilde{F}_k$. Equation (1.1.12) is understood in the next weak sense

$$\left\langle \frac{d}{dt}L_k v_k(t), \phi \right\rangle + \langle L_k v_k, \mathbf{A}_k^*\phi\rangle = \left\langle \psi_k(t), ((\theta_k + \tilde{F}_k)D_k)^*\phi \right\rangle,\ \forall \phi \in \mathcal{D}(\mathbf{A}_k^*),$$

where the dual

$$((\theta_k + \tilde{F}_k)D_k)^*\xi = \nu\xi'''(1), \tag{1.1.13}$$

for all $\xi \in H^4(0,1), \xi(0) = \xi(1) = 0, \xi'(0) = \xi'(1) = 0$.

1.1.3 Further properties of the operators $\mathbf{A}_k$ and D_k

Appealing to Fredholm theory for compact operators, by Lemma 1.1.1, $-\mathbf{A}_k$ has a countable set of eigenvalues $\left\{\lambda_j^k\right\}_{j=1}^{\infty}$; moreover, there is a finite number N_k of eigenvalues λ_j^k for which $\Re\lambda_j^k \geq 0$, the unstable eigenvalues. We denote by $\left\{\phi_j^k\right\}_{j=1}^{\infty}$ and $\left\{\phi_j^{k*}\right\}_{j=1}^{\infty}$ the eigenfunctions of $-\mathbf{A}_k$ and $-\mathbf{A}_k^*$, respectively.

We have a result of "unique continuation" type for the eigenfunctions of the dual operator $-\mathbf{A}_k^*$.

Lema 1.1.2 *Let $\overline{\lambda_j^k}$, an unstable eigenvalue for the dual $-\boldsymbol{A}_k^*$. Then, we can assume that the corresponding eigenfunction ϕ_j^{k*} can be choosen such that $\Re(\phi_j^{k*})'''(1) > 0$.*

Also, we have the next continuity result

Proposition 1.1.1 *For all $0 < |k| \leq S$, D_k is continuous form $\mathbb{C}$ to H.*

1.1.4 Feedback stabilization for the equivalent system (1.1.12)

For simplicity, we shall omit the symbol ˜ and set

$$z_k := L_k v_k,\ \mathbf{B}_k := (\theta_k + \tilde{F}_k)D_k. \tag{1.1.14}$$

Equation (1.1.12) becomes

$$\begin{cases} \frac{d}{dt}z_k(t) + \mathbf{A}_k z_k(t) = \mathbf{B}_k\psi_k(t),\ t > 0, \\ z_k(0) = z_{0k}, \end{cases} \tag{1.1.15}$$

where $z_{0k} = L_k v_k^0$.

We denote by $X_{N_k}^u := linspan\left\{\phi_j^k\right\}_{j=1}^{N_k}$ and by $X_{N_k}^s := linspan\left\{\phi_j^k\right\}_{j=N_k+1}^{\infty}$. Then

$$H = X_{N_k}^u \oplus X_{N_k}^s,$$

as algebric summ. Introduce the projection $P_{N_k} : H \to X^u_{N_k}$, and its adjoint $P^*_{N_k}$, defined as

$$P_{N_k} := -\frac{1}{2\pi i}\int_\Gamma (\lambda I + \mathbf{A}_k)^{-1} d\lambda \text{ and } P^*_{N_k} := -\frac{1}{2\pi i}\int_{\overline{\Gamma}} (\lambda I + \mathbf{A}^*_k)^{-1} d\lambda.$$

Aslo, denote by

$$-\mathbf{A}^u_{N_k} := P_{N_k}(-\mathbf{A}_k) \text{ and } -\mathbf{A}^s_{N_k} := (I - P_{N_k})(-\mathbf{A}_k), \tag{1.1.16}$$

It follows immediately that $-\mathbf{A}^s_{N_k}$ satisfies the next exponential decay on $X^s_{N_k}$

$$\|e^{-t\mathbf{A}^s_{N_k}}\|_{L(H,H)} \le C_{\alpha_0} e^{-\alpha_0 t},\ \forall t \ge 0, \tag{1.1.17}$$

for some $0 < \alpha_0 < |\Re\lambda_{N_k+1}|$.

Next, we decompose system (1.1.15) as

$$z_k = z_{N_k} + \zeta_{N_k},\ \text{where } z_{N_k} := P_{N_k} z_k \text{ and } \zeta_{N_k} := (I - P_{N_k}) z_k,$$

by applying the operators P_{N_k} and $I - P_{N_k}$ to the system (1.1.15), we get

$$\text{on } X^u_{N_k} : \begin{cases} \frac{d}{dt} z_{N_k} + \mathbf{A}^u_{N_k} z_{N_k} = P_{N_k}(\mathbf{B}_k \psi_k), \\ z_{N_k}(0) = P_{N_k} z_{0k}, \end{cases} \tag{1.1.18}$$

$$\text{on } X^s_{N_k} : \begin{cases} \frac{d}{dt} \zeta_{N_k} + \mathbf{A}^s_{N_k} \zeta_{N_k} = (I - P_{N_k})(\mathbf{B}_k \psi_k), \\ \zeta_{N_k}(0) = (I - P_{N_k}) z_{0k}, \end{cases} \tag{1.1.19}$$

respectively.

Because of the relation (1.1.17), system (1.1.19) is stable. Therefore, it remains to stabilize the finite-dimensional unstable system (1.1.18). We have

Lema 1.1.3 *For all $0 < |k| \le S$, there exist $\alpha_1, C_{\alpha_1} > 0$ and a control ψ_k such that, once inserted into the system (1.1.18), the corresponding solution z_{N_k} satisfies*

$$\|z_{N_k}(t)\| \le C_{\alpha_1} e^{-\alpha_1 t} \|z_{0k}\|,\ \forall t \ge 0.$$

Besides, the control can be choosen of class C^1, such that

$$\left|\frac{d}{dt}\psi_k(t)\right| + |\psi_k(t)| \le C_{\alpha_1} e^{-\alpha_1 t} \|z_{0k}\|,\ \forall t \ge 0.$$

Using this result, we get

Theorem 1.1.1 *For all $0 < |k| \le S$, there exists a control ψ_k such that, once inserted into the system (1.1.15), the corresponding solution z_k and the control satisfies the exponential decay*

$$\left|\frac{d}{dt}\psi_k(t)\right| + |\psi_k(t)| \le C_{\alpha_1} e^{-\alpha_1 t} \|z_{0k}\|, \|z_k(t)\| \le C_{\alpha_0} e^{-\alpha_0 t} \|z_{0k}\|, \forall t \ge 0.$$

We derive a feedback form of the controller ψ_k in Theorem 1.1.1 by using a classical approach: minimization of a cost function associated to the linear system (1.1.15), considered into the dual space $X = (H^2(0,1) \cap H_0^1(0,1))^*$.

Theorem 1.1.2 *For all $0 < |k| \leq S$, there is a feedback control*

$$\psi_k = -\nu(L_k^{-2} R_k z_k)'''(1)$$

such that, once inserted into the system (1.1.15)*, the corresponding solution of the closed loop system* (1.1.15)*, satisfies*

$$\|L_k^{-1} z_k(t)\| \leq Ce^{-\gamma t} \|L_k^{-1} z_{0k}\|, \ \forall t \geq 0,$$

for some $C_{\gamma_k}, \gamma_k > 0$. Here $R_k \in L(X, X)$ is a linear self-adjoint operator such as

(i) $R_k : H \to H$,

(ii) R_k *satisfies the next algebraic Riccati type equation*

$$\left\langle L_k^{-1} R_k z_{0k}, L_k^{-1} \boldsymbol{A}_k z_{0k} \right\rangle + \frac{1}{2}\nu^2 |(L_k^{-2} R_k z_{0k})'''(1)|^2 = \frac{1}{2} \|L_k^{-1} z_{0k}\|^2, \forall z_{0k} \in H.$$

1.1.5 Feedback stabilization for the linearized system (1.1.2)

The main result of this section is

Theorem 1.1.3 *The feedback controller*

$$\Psi(t, x) = -\nu \sum_{0<|k|\leq S} (L_k^{-2} R_k L_k v_k(t))'''(1) e^{\mathrm{i}kx}, \tag{1.1.20}$$

where

$$v_k(t, y) = \int_0^{2\pi} v(t, x, y) e^{-\mathrm{i}kx} dx, \ 0 < |k| \leq S,$$

once inserted into equation (1.1.2)*, the correponding solution to the closed-loop system* (1.1.2) *satisfies*

$$\|(u(t), v(t))\|^2 \leq C_\alpha e^{-\alpha t} \|(u^0, v^0)\|^2, \ t \geq 0,$$

for some $C_\alpha, \alpha > 0$.

1.2 Tangential feedback stabilization of periodic fluids in a two-dimensional channel

We consider again the Navier-Stokes equations in a two-dimensional channel, but now with the boundary control acting to the tangential component of the velocity field, on the upper wall.

$$\begin{cases} u_t - \nu \Delta u + u u_x + v v_y = p_x, \ x \in \mathbb{R}, \ y \in (0,1), \\ v_t - \nu \Delta v + u v_x + v v_y = p_y, x \in \mathbb{R}, \ y \in (0,1), \\ u_x + v_y = 0, \\ u(t, x, 0) = 0, \ u(t, x, 1) = \Psi(t, x), \ v(t, x, 0) = v(t, x, 1) = 0, \\ u(t, x + 2\pi, y) = u(t, x, y), \ v(t, x + 2\pi, y) = v(t, x, y), \\ p(t, x + 2\pi, y) = p(t, x, y), \ \forall t \geq 0, \forall x \in \mathbb{R}, \forall y \in (0,1). \end{cases} \tag{1.2.1}$$

Again, the effort is to stabilize the Poiseuille profile, from the previous section. Thus, as before, we are lead to the study of the null stabilization of the system

$$\begin{cases} u_t - \nu\Delta u + u_x U^e + vU^e_y + uu_x + vu_y = p_x, \ x \in \mathbb{R}, y \in (0,1), \\ v_t - \nu\Delta v + v_x U^e + uv_x + vv_y = p_y, \ x \in \mathbb{R}, y \in (0,1), \\ u_x + v_y = 0, \\ u(t,x,0) = 0, \ u(t,x,1) = \Psi(t,x), \ v(t,x,0) = v(t,x,1) = 0, \\ u(t,x+2\pi,y) = u(t,x,y), \ v(t,x+2\pi,y) = v(t,x,y), \\ p(t,x+2\pi,y) = p(t,x,y), \ \forall t \geq 0, \forall x \in \mathbb{R}, \forall y \in (0,1), \end{cases} \tag{1.2.2}$$

with its linearized given by

$$\begin{cases} u_t - \nu\Delta u + u_x U^e + vU^e_y = p_x, \\ v_t - \nu\Delta v + v_x U^e = p_y, \\ u_x + v_y = 0, \\ u(t,x,0) = 0, \ u(t,x,1) = \Psi(t,x), \ v(t,x,0) = v(t,x,1) = 0, \\ u(t,x+2\pi,y) = u(t,x,y), \ v(t,x+2\pi,y) = v(t,x,y), \\ p(t,x+2\pi,y) = p(t,x,y), \ \forall t \geq 0, \forall x \in \mathbb{R}, \forall y \in (0,1). \end{cases} \tag{1.2.3}$$

Arguing as in the first section, we obtain the next stabilization result for the linearized system (1.2.3)

Theorem 1.2.1 *The feedback controller*

$$\Psi(t,x) = -\nu \sum_{0<|k|\leq S} \frac{1}{\mathrm{i}k} (L_k^{-2} \mathcal{R}_k L_k v_k(t))''(1) e^{\mathrm{i}kx}, \tag{1.2.4}$$

where

$$v_k(t,y) = \int_0^{2\pi} v(t,x,y) e^{-\mathrm{i}kx} dx, 0 < |k| \leq S$$

once inserted into the system (1.2.3), the corresponding solution to the closed-loop system (1.2.3) satisfies the exponential decay

$$\|(u(t), v(t))\|^2 \leq C_\beta e^{-\beta t} \|(u^0, v^0)\|^2, \ t \geq 0,$$

for some $C_\beta, \beta > 0$.

1.2.1 Local feedback stabilization for the full nonlinear Navier-Stokes system (1.2.2)

Because of the tangential conditions, we are able in this case to prove also the local stabilization of the nonlinear system (1.2.2), using the fixed point method developed by Barbu and Triggiani [13].

Theorem 1.2.2 *Let* $W := \mathcal{D}(A^{\frac{1}{4}})$ *and the neighbourhood of zero*

$$\mathcal{U}_\rho := \left\{ (u^0, v^0) \in W; \|(u^0, v^0)\|_W \leq \rho \right\}.$$

The feedback controller

$$\Psi(t,x) = -\nu \sum_{0<|k|\leq S} \frac{1}{ik}(L_k^{-2} R_k L_k v_k(t))''(1)e^{ikx},$$

once inserted into the system (1.2.2) implies the existence of a sufficiently small $\rho > 0$ *such that for all initial data* $(u^0, v^0) \in U_\rho$ *there exists a unique solution*

$$(u,v) \in C([0,\infty); W) \cap L^2(0,\infty; Z),$$

of the closed-loop systema (1.2.2)*, which satisfies*

$$\|(u(t),v(t))\|_W \leq Me^{-\omega t}\|(u^0,v^0)\|_W, \forall t > 0,$$

where $Z := \mathcal{D}(A^{\frac{3}{4}})$*. A is the Stokes operator*

$$A = -P\Delta,\ \mathcal{D}(A) = H^2(Q) \cap H_0^1(Q) \cap H,$$

where P *is the Leray projector.*

1.3 Normal feedback stabilization of the periodic flows in a three-dimensional channel

The 2-D results presented above can be extended straightforward to the three-dimensional case. These results are obtained by the author in [67].

1.3.1 Presenting of the problem

The Navier-Stokes equations in a three-dimensional channel are given by

$$\begin{cases} u_t - \nu\Delta u + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{\partial p}{\partial x}, \\ v_t - \nu\Delta v + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{\partial p}{\partial y}, \\ w_t - \nu\Delta w + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z}, \\ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0,\ x, z \in \mathbb{R}, y \in (0,1), t \geq 0, \\ u(t, x+2\pi, y, z+2\pi) = u(t,x,y,z), \\ v(t, x+2\pi, y, z+2\pi) = v(t,x,y,z), \\ w(t, x+2\pi, y, z+2\pi) = w(t,x,y,z), \\ p(t, x+2\pi, y, z+2\pi) = p(t,x,y,z), \\ u(t,x,0,z) = u(t,x,1,z) = 0, \\ v(t,x,0,z) = 0, v(t,x,1,z) = \Psi(t,x,z), \\ w(t,x,0,z) = w(t,x,1,z) = 0, \\ \forall x, z \in \mathbb{R},\ y \in (0,1),\ t \geq 0. \end{cases} \tag{1.3.1}$$

Again the effort is to stabilize the parabolic Poiseuille profile

$$U^e = -\frac{a}{2\nu}(y^2 - y),\ V^e \equiv 0,\ W^e \equiv 0,\ y \in [0,1]. \tag{1.3.2}$$

Decomposing the linearized system of (1.3.1), around the steady-state solution, in Fourier modes, reducing the pressure and using the divergence free condition we obtain

$$\begin{cases} [v''_{kl} - (k^2+l^2)v_{kl}]_t - \nu v''''_{kl} + [2\nu(k^2+l^2) + \mathrm{i}kU^e]v''_{kl} \\ \qquad - [\nu(k^2+l^2)^2 + \mathrm{i}k(k^2+l^2)U^e + \mathrm{i}k(U^e)'']v_{kl} = 0, \\ t \geq 0, y \in (0,1), \\ v'_{kl}(0) = v'_{kl}(1) = 0, v_{kl}(0) = 0, v_{kl}(1) = \psi_{kl}(t). \end{cases} \tag{1.3.3}$$

We immediately notice the similarities between this system and (1.1.4). Therefore, arguing as before and introducing the operators $L_{kl} : \mathcal{D}(L_{kl}) \subset H \to H$, $F_{kl} : \mathcal{D}(F_{kl}) \subset H \to H$ and $\mathbf{A}_{kl} : \mathcal{D}(\mathbf{A}_{kl}) \subset H \to H$, defined as

$$L_{kl}v := -v'' + (k^2+l^2)v, \mathcal{D}(L_{kl}) = H^1_0(0,1) \cap H^2(0,1), \tag{1.3.4}$$

$$F_{kl}v := \nu v'''' - (2\nu(k^2+l^2) + \mathrm{i}kU^e)v'' + (\nu(k^2+l^2)^2 + \mathrm{i}k(k^2+l^2)U^e + \mathrm{i}k(U^e)'')v, \tag{1.3.5}$$

$$\mathcal{D}(F_{kl}) = H^4(0,1) \cap H_0^2(0,1),$$
$$\mathbf{A}_{kl} := F_{kl}L_{kl}^{-1},\ \mathcal{D}(\mathbf{A}_{kl}) = \left\{v \in H : L_{kl}^{-1}v \in \mathcal{D}(F_{kl})\right\}, \tag{1.3.6}$$

respectively, we get

Theorem 1.3.1 *The feedback controler*

$$\Psi(t,x,z) = \sum_{k,l\in\mathbb{Z}} \psi_{kl}(t)e^{ikx}e^{ilz}, \tag{1.3.7}$$

where

$$\psi_{kl}(t) = \begin{cases} -\nu(L_k^{-2}R_kL_kv_{k0}(t))'''(1) & \text{for } 0<|k|\le S,\ l=0, \\ 0 & \text{for } |k|>S,\ l=0, \\ 0 & \text{for } k=0,\ l\in\mathbb{Z}, \\ -\nu(L_{kl}^{-2}R_{kl}L_{kl}v_{kl}(t))'''(1) & \text{for } k,l\in\mathbb{Z}^* \text{ și } \sqrt{k^2+l^2}\le S, \\ 0 & \text{for } k,l\in\mathbb{Z}^* \text{ și } \sqrt{k^2+l^2}>S, \end{cases} \tag{1.3.8}$$

once inserted into the linearized system of (1.3.1), yields the exponential decay for the corresponding solution

$$\|(u(t),v(t),w(t))\|^2 \le C_\gamma e^{-\gamma t}\|(u^0,v^0,w^0)\|^2,\ t\ge 0,$$

for some $C_\gamma, \gamma > 0$.

1.4 Normal feedback stabilization of a MHD flow in a two-dimensional and three-dimensional channel

In this last section, we treat the case when the fluid is electrically conductive and an external magnetic field is applied transversal to the channel. These results are obtained by the author in [70].

1.4.1 Main results

The governing equations of the dynamics are the MHD equations, a combination between the Navier-Stokes equations and Maxwell equations, i.e.,

$$\frac{\partial \mathbf{v}}{\partial t} - \nu\Delta\mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} - \mathcal{N}(\mathbf{j}\times\mathbf{B}) = -\nabla p, \tag{1.4.1}$$

and

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{v}\times\mathbf{B}) + \frac{1}{Rm}\Delta\mathbf{B}, \tag{1.4.2}$$

where $\mathbf{v}$ is the velocity field, $\mathbf{B}$ is the magnetic field, $\mathbf{j}$ is the current density and p is the pressure ν, Rm and $\mathcal{N}$ are the viscosity coefficient, the magnetic Reynolds number and the Stuart number, respectively. We shall consider the fluid with low values of the magnetic

Reynolds numeber. It tourns out that the MHD equations (1.4.1)-(1.4.2) transform into the so-called simplified MDH equations, given by

$$\begin{cases} u_t - \nu\Delta u + \mathcal{N}u + uu_x + vu_y = -p_x, \\ v_t - \nu\Delta v + uv_x + vv_y = -p_y, \\ u_x + v_y = 0, \\ t \geq 0, x \in \mathbb{R}, y \in (0,1), \end{cases} \tag{1.4.3}$$

in 2-D,

$$\begin{cases} U_t - \nu\Delta U + UU_x + VU_y + WU_z - \mathcal{N}\phi_z + \mathcal{N}U = -P_x, \\ V_t - \nu\Delta V + UV_x + VV_y + WV_z = -P_y \\ W_t - \nu\Delta W + UW_x + VW_y + WW_z + \mathcal{N}\phi_x + \mathcal{N}W = -P_z, \\ \Delta\phi = W_x - U_z, \\ U_x + V_y + W_z = 0, \\ t \geq 0, x, z \in \mathbb{R}, y \in (0,1), \end{cases} \tag{1.4.4}$$

in 3-D, respectively.

The steady-state solutions which are stabilized are the Hartmann-Poiseuille profiles, given by

$$U^e = \frac{\sinh\left(\sqrt{\frac{1}{\nu}\mathcal{N}}(1-y)\right) - \sinh\sqrt{\frac{1}{\nu}\mathcal{N}} + \sinh\left(\sqrt{\frac{1}{\nu}\mathcal{N}}y\right)}{2\sinh\left(\sqrt{\frac{1}{\nu}\mathcal{N}}/2\right) - \sinh\sqrt{\frac{1}{\nu}\mathcal{N}}}, \ V^e \equiv 0, \tag{1.4.5}$$

in 2-D,

$$U^e = \frac{\sinh\left(\sqrt{\frac{1}{\nu}\mathcal{N}}(1-y)\right) - \sinh\sqrt{\frac{1}{\nu}\mathcal{N}} + \sinh\left(\sqrt{\frac{1}{\nu}\mathcal{N}}y\right)}{2\sinh\left(\sqrt{\frac{1}{\nu}\mathcal{N}}/2\right) - \sinh\sqrt{\frac{1}{\nu}\mathcal{N}}}, \ V^e \equiv 0, \ W^e \equiv 0, \tag{1.4.6}$$

in 3-D, respectively.

Let us notice the clear similarities between the SMHD equations and the ones studied in the previous sections. One might suspect that, arguing as before, one can obtain a normal feedback stabilization result as before. This is indeed, making use of the Fourier functional setting, one can argue as before in order to obtain the wanted results.

Chapter 2

Internal stabilization of Navier-Stokes equations with exact controllability on spaces with finite co-dimensiona

In this chapter, we design an internal stabilizing feedback controller for the Navier-Stokes equations. Besides, we show that this controller steers the initial data into the space of stable modes, in finite time. These results were obtained by the author in [21], jointly with V. Barbu.

2.1 Presentation of the problem

The Navier-Stokes equations with null boundary conditions are

$$\begin{cases} \frac{\partial \mathbf{v}}{\partial t} - \nu\Delta\mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} = f^e + \nabla p \text{ in } (0,\infty)\times\mathcal{O}, \\ \nabla\cdot\mathbf{v} = 0 \text{ in } (0,\infty)\times\mathcal{O}, \\ \mathbf{v} = 0 \text{ on } (0,\infty)\times\partial\mathcal{O}, \\ \mathbf{v}(0) = \mathbf{v}_o \text{ in } \mathcal{O}, \end{cases} \tag{2.1.1}$$

where $\mathcal{O}\subset\mathbb{R}^d,\ d=2,3$, is an open domain with smooth boundary $\partial\mathcal{O}$.

We consider a steady-state solution $\mathbf{v}^e=\mathbf{v}^e(x)$ of N-S equations, i.e.,

$$\begin{cases} -\nu\Delta\mathbf{v}^e + (\mathbf{v}^e\cdot\nabla)\mathbf{v}^e = f^e + \nabla p^e \text{ în } \mathcal{O}, \\ \nabla\cdot\mathbf{v}^e = 0 \text{ în } \mathcal{O};\ \mathbf{v}^e = 0 \text{ pe } \partial\mathcal{O}. \end{cases} \tag{2.1.2}$$

Let $\mathcal{O}_0 \subset \mathcal{O}$, we associate to the system (2.1.1) the next internal control problem

$$\begin{cases} \frac{\partial \mathbf{v}}{\partial t} - \nu\Delta\mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} = f^e + \nabla p + m\Psi \text{ in } (0,\infty)\times\mathcal{O}, \\ \nabla\cdot\mathbf{v} = 0 \text{ in } (0,\infty)\times\mathcal{O}, \\ \mathbf{v} = 0 \text{ on } (0,\infty)\times\partial\mathcal{O}, \\ \mathbf{v}(0) = \mathbf{v}_o \text{ in } \mathcal{O}, \end{cases} \tag{2.1.3}$$

where m is the characterisitc function of the set $\mathcal{O}_0$ and Ψ is the control.

2.2 Main results

We set

$$H_\pi := \left\{\mathbf{v} \in \left(L^2(\mathcal{O})\right)^d : \nabla\cdot\mathbf{v} = 0 \text{ in } \mathcal{O},\ \mathbf{v}\cdot\mathbf{n} = 0 \text{ on } \partial\mathcal{O}\right\}, \tag{2.2.1}$$

where $\mathbf{n}$ is the unit outward normal to the boundary $\partial\mathcal{O}$. The Leray projector is defined as $P : (L^2(\mathcal{O}))^d \to H_\pi$. We introduce also

$$A := -P\Delta,\ \mathcal{D}(A) = \left(H_0^1(\mathcal{O}) \cap H^2(\mathcal{O})\right)^d \cap H_\pi,$$

$$\mathcal{A}\mathbf{v} := \nu A\mathbf{v} + P((\mathbf{v}\cdot\nabla)\mathbf{v}^e + (\mathbf{v}^e\cdot\nabla)\mathbf{v}),\ \mathcal{D}(\mathcal{A}) = \mathcal{D}(A),$$

and

$$B\mathbf{v} := P((\mathbf{v}\cdot\nabla)\mathbf{v}),\ \mathbf{v}\in\mathcal{D}(A).$$

Thus, redefining $\mathbf{v} := P(\mathbf{v} - \mathbf{v}^e)$, we rewrite the system (2.1.3) as

$$\begin{cases} \frac{d}{dt}\mathbf{v} + \mathcal{A}\mathbf{v} + B\mathbf{v} = P(m\Psi) \text{ in } (0,\infty)\times\mathcal{O}, \\ \mathbf{v}(0) = \mathbf{v}_o := \mathbf{v}_o - \mathbf{v}^e, \end{cases} \tag{2.2.2}$$

since by applying the Leray projector we reduce the pressure.

Next, we introduce the feedback controlle

$$\Psi(t) := -\eta \sum_{j=1}^N sign\left(\langle P_N\mathbf{v}(t), \phi_j^*\rangle\right) P_N\Phi_j, \tag{2.2.3}$$

where $\eta \in \mathbb{R}_+$, $sign$ is the multivalued function on $\mathbb{C}$, defined as

$$sign(z) := \begin{cases} \frac{z}{|z|}, & \text{if } z \neq 0, \\ \{w\in\mathbb{C} : |w| \leq 1\}, & \text{if } z = 0, \end{cases} \tag{2.2.4}$$

and $\Phi_j \in H_\pi$ are defined as

$$\Phi_j := \sum_{k=1}^N \alpha_{jk}\phi_k^*,\ j = 1, ..., N,$$

with

$$\sum_{k=1}^{N} \alpha_{ik} \left\langle \phi_k^*, \phi_j^* \right\rangle_0 = \delta_{ij},\ i,j = 1,...,N. \tag{2.2.5}$$

Here

$$\langle \phi, \psi \rangle_0 := \int_{\mathcal{O}} m\phi\overline{\psi}d\xi, \forall \phi, \psi \in (L^2(\mathcal{O}))^d.$$

We have the next result of stabilization for the N-S system

Theorem 2.2.1 *Let $T, \rho > 0$ sufficiently small. For all $\mathbf{v}_o \in W$, such tha $\|\mathbf{v}_o\|_W \leq \rho$, the problem*

$$\begin{cases} \dfrac{d\mathbf{v}}{dt} + \mathcal{A}\mathbf{v} + \eta \displaystyle\sum_{j=1}^{N} sign(\left\langle P_N\mathbf{v}, \phi_j^* \right\rangle) P_N m(\Phi_j) + B\mathbf{v} = 0,\ t \geq 0, \\ \mathbf{v}(0) = \mathbf{v}_o, \end{cases} \tag{2.2.6}$$

is well-posed on W with unique solution

$$\mathbf{v} \in C([0, \infty); W) \cap L^2(0, \infty; Z),$$

provided that η is such that

$$\eta \geq \max \left\{ \frac{\Re\lambda_j \left(k\|\phi_j^*\| + \rho \right)}{e^{\Re\lambda_j T} - 1}; j = 1, ..., N \right\}. \tag{2.2.7}$$

Besides, these solutions satisfy

$$P_N\mathbf{v}(t) = 0, \forall t \geq T, \tag{2.2.8}$$

and

$$\|\mathbf{v}(t)\| \leq Ce^{-\beta t}\|\mathbf{v}_0\|, \forall t \geq T. \tag{2.2.9}$$

Here

$$W := (H^{\frac{1}{2}-\epsilon}(\mathcal{O}))^d \cap H_\pi \text{ and } Z := (H^{\frac{3}{2}-\epsilon}(\mathcal{O}))^d \cap H_\pi$$

for $d = 2$,

$$W := (H^{\frac{1}{2}+\epsilon}(\mathcal{O}))^d \cap H_\pi \text{ and } Z := (H^{\frac{3}{2}+\epsilon}(\mathcal{O}))^d \cap H_\pi$$

for $d = 3$.

Chapter 3

Internal stabilization of a finite set of stedy-state solutions to the Na-vier-Stokes equations

In this chapter, we shall design a control which stabilizes, not one steady-state solution, but a finte set of steady-state solutions to the N-S system. The results presented below were obtained by the author in [69].

3.1 Main results

Remember the controlled N-S equations with zero boundary conditions

$$\begin{cases} \mathbf{v}_t(x,t) - \nu\Delta\mathbf{v}(x,t) + (\mathbf{v}\cdot\nabla)\mathbf{v}(x,t) \\ \qquad\qquad = m(x)\Psi(x,t) + f^e(x) + \nabla p(x,t), \text{ in } Q = \mathcal{O}\times(0,\infty), \\ \nabla\cdot\mathbf{v} = 0, \text{ in } Q, \\ \mathbf{v} = 0, \text{ on } \Sigma = \partial\mathcal{O}\times(0,\infty), \\ \mathbf{v}(x,0) = \mathbf{v}_0(x), \text{in } \mathcal{O}. \end{cases} \tag{3.1.1}$$

Applying the Leray projector, these can be rewritten as

$$\frac{d\mathbf{v}}{dt} + \nu A\mathbf{v} + B\mathbf{v} = P(mu + f^e); \mathbf{v}(0) = \mathbf{v}_0 \in H_\pi. \tag{3.1.2}$$

The main result of internal stabilization in [10], says that there exists a neighbourhood U_ρ of zero and a feedback controller which stabilzes system (3.1.2) in it. Moreover, we know that N-S obeys the next generic property: for "allmost all" external forces f_e, the number of stedy-state solutions is finite. Thus, let $\{\mathbf{v}_1^e, \mathbf{v}_2^e, ..., \mathbf{v}_N^e\}$ the steady-state solutions. For each $\mathbf{v}_i^e, i = 1, ..., N$, there is a feedback controller $\Psi_i = \Psi_i(\mathbf{v} - \mathbf{v}_i^e), i = 1, ..., N$, such that the solution to

$$\frac{d\mathbf{v}}{dt} + \nu A\mathbf{v} + B\mathbf{v} = P(m\Psi_i(\mathbf{v} - \mathbf{v}_i^e)) + Pf_e, \ t \geq 0; \mathbf{v}(0) = \mathbf{v}_0, \tag{3.1.3}$$

satisfies

$$|\mathbf{v}(t)-\mathbf{v}_i^e|_{\frac{1}{2}} \leq C_i e^{-\gamma_i t}|\mathbf{v}_0-\mathbf{v}_i^e|_{\frac{1}{2}}, t\geq 0, \tag{3.1.4}$$

for $\mathbf{v}_0 \in U_{\rho_i}$. Here we denote by $|\mathbf{v}|_{\frac{1}{2}} := \|A^{\frac{1}{4}}\mathbf{v}\|, \forall \mathbf{v}\in \mathcal{D}(A^{\frac{1}{4}})$.

Consider the sets

$$\mathcal{U}_i=\left\{\mathbf{v}_0\in H_\pi; |\mathbf{v}_0-\mathbf{v}_i^e|_{\frac{1}{2}} < \frac{\rho_i}{C_i}\right\}, i=1,...,N. \tag{3.1.5}$$

and $\epsilon>0$ such that

$$\left\{\mathbf{v}; |\mathbf{v}-\mathbf{v}_i^e|_{\frac{1}{2}} < (1+\epsilon)\rho_i\right\} \cap \left\{\mathbf{v}; |\mathbf{v}-\mathbf{v}_j^e|_{\frac{1}{2}} < (1+\epsilon)\rho_j\right\} = \emptyset, \tag{3.1.6}$$

$\forall j\neq i, i, j=1,...,N$.

Introducce the function $w:\mathbb{R}_+\to[0,1]$, defined as

$$w(r)=\left\{\begin{array}{cc} 1 & 0\leq r\leq 1,\\ 0 & r\geq 1+\epsilon,\\ \text{netedă} & 1<r<1+\epsilon. \end{array}\right.$$

Finally, introduce $\chi_i:\mathcal{D}(A^{\frac{1}{4}})\to[0,1]$, as

$$\chi_i(\mathbf{v})=w\left(\frac{|\mathbf{v}|_{\frac{1}{2}}}{\rho_i}\right), \forall \mathbf{v}\in\mathcal{D}(A^{\frac{1}{4}}), i=1,...,N. \tag{3.1.7}$$

Then, the controller

$$\Psi(\mathbf{v}) := \sum_{i=1}^{N} \chi_i(\mathbf{v}-\mathbf{v}_i^e)\Psi_i(\mathbf{v}). \tag{3.1.8}$$

stabilizes the finite-set of steady-state solutions $\mathbf{v}_i^e$.

Theorem 3.1.1 *The feedback controller Φ, defined by* (3.1.8)*, exponentially stabilizes every stationary solution $\mathbf{v}_i^e, i=1,...,N$ in the neighbourhood $\mathcal{U}_i, i=1,...,N$. More precisely, for all $\mathbf{v}_0\in\mathcal{U}_i, i=1,...,N$ there exists a unique weak solution*

$$\mathbf{v}\in L^\infty(0,T;H_\pi)\cap L^2(0,T;V), \frac{d\mathbf{v}}{dt}\in L^{\frac{4}{3}}(0,T;V*),$$

for $d=3$, and

$$\mathbf{v}\in L^\infty(0,T;H_\pi)\cap L^2(0,T;V), \frac{d\mathbf{v}}{dt}\in L^{2}(0,T;V*)$$

for $d=2$, $\forall T>0$, to the closed loop system

$$\frac{d\mathbf{v}}{dt}+\nu A\mathbf{v}+B\mathbf{v}=P(m\sum_{j=1}^{N}\chi_j(\mathbf{v}-\mathbf{v}_j^e)\Psi_j(\mathbf{v}))+Pf^e,\ t\geq 0; \mathbf{v}(0)=\mathbf{v}_0, \tag{3.1.9}$$

whihc satisfies the exponential decay

$$|\mathbf{v}(t)-\mathbf{v}_i^e|_{\frac{1}{2}} \leq C_i e^{-\gamma_i t}|\mathbf{v}_0-\mathbf{v}_i^e|_{\frac{1}{2}}, t\geq 0.$$

Chapter 4

Apendix

This chapter briefly presents the main theoretical results used during the presentation. More precisely, we remember results about compact opertors in Banach spaces, spectral decomposition, Fredholm theorey, the extended of an operator, semigroup theory and dynamical systems theorey.

Bibliography

[1] Adams, D., *Sobolev spaces,* Academic Press (1975).

[2] Baker, J., Armaou, A. şi Christofides, P., *Nonlinear control of incompressible fluid flow: application to Burgers equation and 2D channel flow,* J. Math. Anal. Appl. **252** (2000), 230-255.

[3] Balogh, A., Liu, W.-J. şi Krstic, M., *Stability enhancement by boundary control in 2-D channel flow,* IEEE Transactions on Automatic Control **46** (2001), 1696-1711.

[4] Balakrishanan, A.V., *Applied functional analysis,* Springer, Berlin (1981).

[5] Barbu, V., *The time optimal control of Navier-Stokes equations,* Systems Control Lett. **30** (1997), 93-100.

[6] Barbu, V., *Partial differential equations and boundary value problems,* Kluwer Academic Publishers (1998).

[7] Barbu, V. şi Sritharan, S.S., *H^∞-control theory of fluid dynamics,* R. Soc. Lond. Proc. Ser. A Math. Phys. Eng. Sci. **454** (1998), no. 1979, 3009-3033.

[8] Barbu, V., *Feedback stabilization of Navier-Stokes equations,* ESAIM COCV **9** (2003), 197-206.

[9] Barbu, V. şi Lefter, C.G., *Internal stabilizability of the Navier-Stokes equations,* Systems and Control Lett. **48** (2003), 161-167.

[10] Barbu, V. şi Triggiani, R., *Internal stabilization of Navier-Stokes equations with finite-dimensional controllers,* Indiana Univ. Math. Journal **53** (2004), 1443-1494.

[11] Barbu, V. şi Lefter, C., *Optimal control of ordinary differential equations,* Handbook of differential equations, Elsevier, Amsterdam (2005).

[12] Barbu, V., Lasiecka, I. şi Triggiani, R., *Abstract settings for tangential boundary stabilization of Navier-Stokes equations by high-and low-gain feedback controllers,* Nonlin. Anal. **64** (2006), 2704-2746.

[13] Barbu, V., Lasiecka, I. şi Triggiani, R., *Tangential boundary stabilization of Navier-Stokes equations,* Memories AMS **851** (2006), pp+128.

[14] Barbu, V., *Stabilization of a plane channel flow by wall normal controllers,* Nonlin. Anal. Theory-Methods Appl. **56** (2007), 145-168.

[15] Barbu, V., *Nonlinear Differential Equations of Monotone Type in Banach Spaces,* Springer, New York (2010).

[16] Barbu, V., *Stabilization of Navier-Stokes Flows,* Springer, New York (2010).

[17] Barbu, V., *Stabilization of a plane periodic channel flow by noise wall normal controllers,* Systems Control Lett. **59** (2010), 608-614.

[18] Barbu, V., Rodrigues, S.S. şi Shirikyan, A., *Internal exponential stabilization to a nonstationary solution for the 3D Navier-Stokes equations,* SIAM J. Control Optim. **49** (2011), no.4, 1454-1478.

[19] Barbu, V. şi Lasiecka, I., *The unique continuation property of eigenfunctions to Stokes-Oseen operator is generic with respect to the coefficients,* Nonlin. Anal. Theory-Methods Appl. http://dx.doi.org/10.1016/j.na.2011.07.056 (2011).

[20] Barbu, V., *Stabilization of Navier-Stokes equations by oblique boundary feedback controllers* http://arxiv.org/abs/arXiv:1106.3931 (2011).

[21] Barbu, V. şi Munteanu, I., *Internal stabilization of Navier-Stokes equation with exact controllability on spaces with finite codimension,* Evol. Eqs. Control Theory **1** (2012), 1-16.

[22] Bedra, M., *Feedback stabilization of the 2D and 3D Navier-Stokes equations based on an extended system,* ESAIM COCV **15** (2009), 934-968.

[23] Bewely, T.R. şi Liu, S., *Optimal and robust control and estimation of linear paths to transition,* J. Fluid Mech. **365** (1998), 305-349.

[24] Bewely, T.R., *New frontiers for control in fluid mechanics: a renaissance approach,* în ASME FEDSM 99-6926 (1999).

[25] Bewely, T.R., Temam, R. şi Ziane, M., *A general framework for robust control in fluid mechanics,* Physica D **138** (2000), 360-392.

[26] Bewely, T.R., Moin, P. şi Temam, R., *DNS-based predictive control of turbulence: an optimal benchmark for feedback algorithms,* J. Fluid Mech. **447** (2001), 179-225.

[27] Cochran, J., Vazquez, R. şi Krstic, M., *Backstepping boundary control of Navier-Stokes channel flow: a 3D extension,* 25th Amer. Control Conf. (2006).

[28] Constantin, P. şi Foias, C., *Navier-Stokes Equations* , Univ. Chicago Press (1989).

[29] Coron, J.-M., *On the controllability of 2-D incompressible perfect fluids,* J. Math. Pures Appl. **75** (1996), 155-188.

[30] Coron, J.-M., *On the controllability of the 2-D incompressible Navier-Stokes equations with the Navier slip boundary conditions,* ESAIM: Control, Optim. Cal. Var. **1** (1996), 35-75.

[31] Coron, J.-M. şi Fursikov, A.V., *Global exact controllability of the 2-D Navier-Stokes equations on a manifold without boundary,* Russian J. Math. Phys. **4** (1996), 429-448.

[32] Coron, J.-M., *On null asymptotic stabilization of the 2-D Euler equation of incompressible fluids on simply connected domains,* SIAM J. Control Optim. **37** (1999), 1874-1896.

[33] Coron, J.-M., *Control and Nonlinearity,* AMS, Providence, RI (2007).

[34] Cortelezzi, L., Speyer, J.L., Lee, K.H. şi Kim, K., *Robust reduced-order control of turbulent channel flows via distributed sensors and actuators,* Proc. 37th IEE Conf. Decision Control, Tampa, Fl. (1998), 1906-1911.

[35] Desai, M. şi Ito, K., *Optimal controls of Navier-Stokes equations,* SIAM J. Control Optim. **32** (1994), 1428-1446.

[36] Fattorini, H.O. şi Sritharan, S.S., *Existence of optimal controls for viscous flow problems,* Proceedings of the Royal Society of London Ser. A **439** (1992), 81-102.

[37] Fattorini, H.O. şi Sritharan, S.S., *Necessary and sufficient conditions for optimal controls in viscous flow problems,* Proceedings of the Royal Society of Edinburgh Ser. A **124A** (1994), 211-251.

[38] Fattorini, H.O. şi Sritharan, S.S., *Optimal chattering controls for viscous flow,* Nonlinear Anal. Theory-Methods Appl. **25** (1995), 763-797.

[39] Fattorini, H.O. şi Sritharan, S.S., *Optimal control problems with state constrains in fluid mechanics and combustion,* Appl. Math. Optim. **38** (1998), 159-192.

[40] Fernandez-Cara, E., *On the approximate and null controllability of the Navier-Stokes equations,* SIAM Rev. **41** (1999), 269-277.

[41] Fortin, A., Jardak, M., Gervais, J.J. şi Pierre, R., *Old and new results on the two-dimensional Poiseuille flow,* J. Comput. Physics **115** (1994), 455-469.

[42] Fursikov, A.V., Imanuvilov, O.Y., *On exact boundary zero-controllability of two-dimensional Navier-Stokes equations- Mathematical problems for Navier-Stokes equations (Centro, 1993),* Acta. Appl. Math. **37** (1994), 67-76.

[43] Fursikov, A.V. şi Imanuvilov, O.Y., *Local exact controllability for the Boussinesques equation,* SIAM J. Control Optimiz. **36** (1998), 391-421.

[44] Fursikov, A.V., Gunzburger, M.D. şi Hou, L.S., *Boundary value problems and optimal boundary control for the Navier-Stokes system: the two-dimensional case,* SIAM J. Control Optim. **36** (1998), 852-894.

[45] Fursikov, A.V., *Real processes of the 3-D Navier-Stokes systems and its feedback stabilization form the boundary,* AMS Translations. Partial Diff. Eqs. M. Vishnik Seminar (2006).

[46] Hartmann, J., *Theory of the laminar flow of an electrically conductive liquid in a homogeneous magnetic field,* Det Kgl. Danske Vidensk-abernes Selskab Mathematisk-fysiske Meddelelser XV **6** (1937), 1-27.

[47] Hou, L.S. şi Yan, Y., *Dynamics and approximations of a velocity tracking problem for the Navier-Stokes flows with picewise distributed control,* SIAM J. Control Optim. **35** (1997), 1847-1885.

[48] Howie, J.M., *Fundamentals of Semigroup Theory,* Clarendon Press (1995).

[49] Imanuvilov, O.Y., *On exact controllability for Navier-Stokes equations,* ESAIM COCV **3** (1998), 97-131.

[50] Jiménez, J., *Transition to turbulence in two-dimensional Poiseuille flow,* J. Fluid Mech. **218** (1990), 265-297.

[51] Joshi, S.S., Speyer, J.L. şi Kim, J., *A system theory approach to the feedback stabilization of infinitesimal and finite-amplitude distrubances in plane Poiseuille flow,* J. Fluid Mech. **332** (1997), 157-184.

[52] Joshi, S.S., Speyer, J.L. şi Kim, J., *Finite-dimensional optimal control of Poiseuille flow,* J. Guid. Control. Dyna. **22** (1999), 340-348.

[53] Kato, T., *Perturbation Theory for Linear Operators,* Die Grundlehren der mathematischen Wissenschaften, Band 132, Springer-Verlag New York, Inc., New York (1966).

[54] Kokotovic, P., Krstic, M. şi Kanellakopoulos, I., *Nonlinear and adaptive control design,* John Wiley and Sons, New York (1995).

[55] Lasiecka, I. şi Triggiani, R., *Control Theory for Partial Differential Equations: Continuous and Approximation Theories: Abstrac Parabolic Systems,* Encyclopedia of Mathematics and its Applications **74**, Cambridge University Press, Cambridge (2002), pp+648.

[56] Lefter, C.G., *Calculul variaţiilor şi controlul sistemelor diferenţiale,* Ed. Al. Myller, Iaşi (2006).

[57] Lefter, C.G., *Ecuaţii diferenţiale şi sisteme dinamice,* Ed. Al. Myller, Iaşi (2006).

[58] Lefter, C.G., *Feedback stabilization of 2D Navier-Stokes equations with Navier slip boundary conditions,* Nonlinear Analysis **70** (2009), 553-562.

[59] Leray, J., *Etude de diverses équations intégrales non linéares et de quelques problémes que pose l'hydrodynamique,* J. Math. Pures Appl. **12** (1933), 1-82.

[60] Leray, J., *Essai sur les mouvements plans d'un liquide visqueux que limitent des parois,* J. Math. Puress Appl. **13** (1934), 331-418.

[61] Leray, J., *Essai sur le mouvement d'un liquide visqueux emplissant l'espace,* Acta Math. **63** (1934), 193-248.

[62] Lock, R.C., *The stability of the flow of an electrically conducting fluid between parallel planes under a transverse magnetic field,* Proceedings of Royal Society of London A **233** (1955), 100- 105.

[63] Lorenzi, A. şi Munteanu, I., *Recovering a constant in the two-dimensional Navier-Stokes system with no initial condition* (trimis spre publicare).

[64] Luo, L. şi Schuster, E., *Mixing enhancement in 3D MHD channel flow by boundary electrical potential,* Amer. Control Conf. (2010), 3347-3352.

[65] Munteanu, I., *Tangential feedback stabilization of periodic flows in a 2-D channel,* Diff. Int. Eqs. **24** (2011), 469-494.

[66] Munteanu, I., *Normal feedback stabilization of periodic flows in a two-dimensional channel,* J. Optimiz. Theory Appl. **152** (2012), 413-443.

[67] Munteanu, I., *Normal feedback stabilization of periodic flows in a three-dimensional channel,* Num. Funct. Anal. Optimiz. **33** (2012), 611-637.

[68] Munteanu, I., *Existence of solutions for models of shallow water in a basin with a degenerate varying bottom,* J. Evol. Eqs. **2** (2012), 413-443.

[69] Munteanu, I., *Internal stabilizable feedback controller for a finite set of equilibrium solutions to the Navier-Stokes equations,* An. Stiint. Univ. Al. I. Cuza, Ser. noua, Mat. (sub tipar).

[70] Munteanu, I., *Normal feedback stabilization and observer design for linearized MHD channel flow at low magnetic Reynolds number* (trimis spre publicare).

[71] Munteanu, I., *Stability in periodic MHD channel flow subject to low external magnetic field* (manuscris).

[72] Pazy, A., *Semigroups of linear operators and applications to partial differential equations,* Springer, Berlin (1985).

[73] Potter, M.C. şi Kutchey, J.A., *Stability of plane Hartmann flow subject to a transverse magnetic field,* Phisics of Fluids **16**(1848).

[74] Ravindran, S.S., *Reduced-order adaptive controllers for fluid flows using POD,* J. Scientific Computing **15** (2000), 457-478.

[75] Raymond, J.P., *Feedback boundary stabilization of the two-dimensional Navier-Stokes equations,* Siam J. Control Optimiz. **45** (2006), 790-828.

[76] Rozhdestvensky, B.L. şi Simakin, I.N., *Secondary flows in a plane channel: their relationship and comparison with turbulent flows,* J. Fluid Mech. **147** (1984), 261-289.

[77] Schuster, E., Luo, L. şi Krstic, M, *MHD channel flow control in 2D: Mixing enhancement by boundary feedback,* Automatica **44** (2008), 2498-2507.

[78] Shirikyan, A., *Exact controllability in projections for three-dimensional Navier-Stokes equations,* Ann. I. H. Poincaré **24** (2007), 521-537.

[79] Smith, B.L. şi Glezer, A., *The formulation and evolution of synthetic jets,* Phys. Fluids **10** (1998), 2281-2297.

[80] Takashima, M., *The stability of the modified plane Pousseuille flow in the presence of a transverse magnetic field,* Fluid Dynamics Research **17** (1996), 293-310.

[81] Temam, R., *Navier-Stokes Equations and Nonlinear Functional Analysis: second edition,* Society for industrial and applied mathematics, Philadelphia (1995).

[82] Temam, R., *Infinite-dimensional dynamical systems in mechanics and physics,* Springer-Verlag, New York (1997).

[83] Triggiani, R., *Stability enhancement of a 2-D linear Navier-Stokes channel flow by a 2-D wall normal boundary controller,* Discrete and Contin. Dyn. Syst. SB, 8279-314.

[84] Vazquez, R. şi Krstic, M., *A closed-form observer for the channel flow Navier-Stokes system,* Proceedings of the 2005 CDC (2005), 5959-5964.

[85] Vazquez, R. şi Krstic, M., *A closed-form feedback controller for stabilization of the linearized 2-D Navier-Stokes Poiseuille flow,* IEEE Trans. Autom. Control **52** (2007), 2298-2312.

[86] Vazquez, R, Schuster, E. şi Krstic, M., *Magnetohydrodynamic state estimation with boundary sensors,* Automatica **44** (2008), 2517-2527.

[87] Vazquez, R, Schuster, E şi Krstic, M., *A closed-form full-state feedback controller for stabilization of 3D magnetohydrodynamic channel flow,* Journal of Dyn. Syst. **131** (2009).

[88] Vladimirov, V. şi Lin, K., *The three-dimensional stability of steady MHD flows of an ideal fluid,* Physics of Plasmas **5** (1998), 4199-4204.

[89] Vrabie, I., C_0-*semigroups and applications,* Ser. Mathematics Studies no. 191, Elsevier, North-Holland, Amsterdam (2003).

[90] Shirikyan, A., *Approximate controllability for three-dimensional Navier-Stokes equations,* Comm, Math. Phys. **266**) (2006), 123-151.

[91] Shirikyan, A., *Exact controllability in projections for three-dimensional Navier-Stokes equations,* Ann. I. H. Poincaré **24** (2007), 521-537.

[92] Xu, C., Schuster, E., Vazquez, R. şi Krstic, M., *Stabilization of linearized 2D magnetohydrodynamic channel flow by backstepping boundary control,* Syst. Control Lett. **57** (2008), 805-812.

[93] Zabczyk, J., *Mathematical control theory: an introduction,* Systems Control: Foundations Applications, Birkh auser Boston Inc., Boston, MA (1992).